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Coauthors: Istvan Kovacs

FROM FINITE TOPOLOGIES AND LATTICES TO THE AUTOMORPHISM GROUPS OF RATIONAL SCHUR RINGS

MIKHAIL KLIN

A Cayley graph Γ over cyclic group $\mathbb{Z}_n$ of order n is called a circulant graph. Graph Γ is called rational if the spectrum of its adjacency matrix consists only of rational (in fact integer) numbers. The main motivation of this presentation stems from our interest to establish efficient procedure to determine automorphism group of a rational circulant graph.

We employ methodology of Schur rings (briefly, S-rings), see [3].

Problem of classification of rational S-rings (S-rings of traces in original Schur-Wielandt's terminology) goes back to the seminal paper of Schur (1933). Various particular cases were considered a few decades ago by Ya. Yu. Gol'fand, Klin and R. Pöschel. In particular, for the case when n is a product of k distinct primes, Gol'fand (1985) established a bijection between rational S-rings over $\mathbb{Z}_n$ and finite topologies on the set of cardinality k.

An elegant description of all rational S-rings over $\mathbb{Z}_n$ was provided by Muzychuk [2] in terms of sublattices of the lattice of all natural divisors of n.

Another origin of our approach stems to the operation of crested product of association schemes [1]. Note that similar concepts for the particular case of S-rings over cyclic groups were suggested by K. H. Leung and S. C. Ma, S. Evdokimov and I. N. Ponomarenko. A more general operation of wedge product of association schemes is treated by Muzychuk.

Merging ideas of our predecessors, we consider simple reduction rules which allow for a given rational S-ring S over $\mathbb{Z}_n$ to describe its automorphism group $Aut(\Gamma)$. Special attention is paid to those particular cases when $Aut(\Gamma)$ appears via iterative use of wreath products and direct products of symmetric groups. For a rational circulant graph Γ we determine such S-ring S that $Aut(\Gamma) = Aut(S)$.

Note that it follows from our results that all rational S-rings over $\mathbb{Z}_n$ are Schurian.

This expository talk is based on the cooperation of the authors with O.H. Kegel and M. Muzychuk.

References

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Ben-Gurion University of the Negev, Beer Sheva, Israel
E-mail address: klin@cs.bgu.ac.il